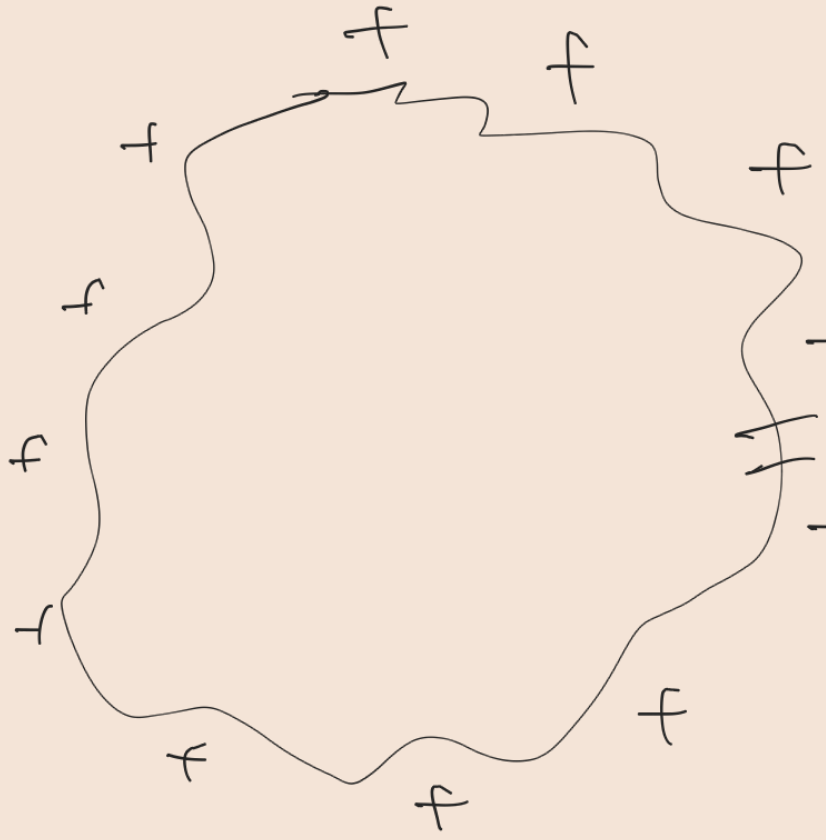
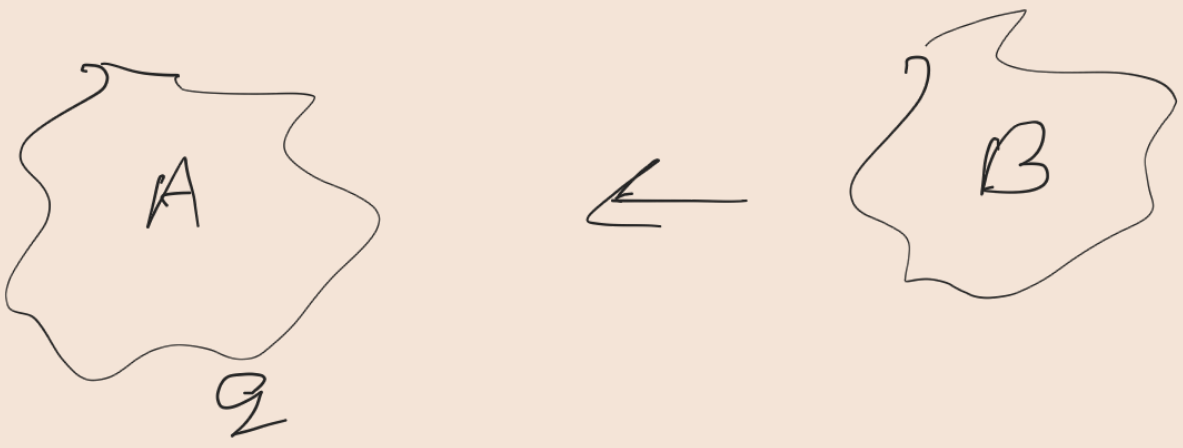
Let find the self energy of an isolated arbitrary conductor, having net charge 'q' & potential 'V'.



If we multiply 'dq' with 'V', we get the interaction of 'dq' with all other charges on the conductor. On summing up (dq)(V) over the entire body, the interaction

between any two charges will hence be counted twice. Hence, self energy of any conductor is $\left(\frac{qV}{2}\right)$. Now the problem is easy.

Step - I,



$$'SE'_A + W_{ext} = \left(\frac{qV}{2}\right)$$

↓ Because 'A & 'B become single conductor.

Also,
$W_{ext} = -W_{field}$

$$W_1 = SE_A - \frac{qV}{2}.$$

Step - 2.

In this situation also, finally 'A' & 'B' become single conductor with net charge 'q'. So potential is 'V'.

$$SE_B - W_2 = \frac{qV}{2}.$$

$$W_2 = SE_B - \frac{qV}{2}.$$

Step - 3

When 'A' & 'B' both are joined with charge 'q'

each, final potential is (2V). Then,

$$(\Delta\varepsilon_A + \Delta\varepsilon_B) - W_{reqd}$$

$$= \frac{(2q)(2V)}{2} = 2qV$$

Required work is

$$W_1 + \frac{qV}{2} + W_2 + \frac{qV}{2} - 2qV$$

$$= \boxed{W_1 + W_2 - qV}$$